

Optimized Medication due to the Law by Weber and Fechner

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Abstract

Although the law by Weber and Fechner is well-known for a long time, there is repeatedly a lack of knowledge about the finding of optimal medication for each singular patient. Especially about the correct reduction and discontinuation of a medicament, which has been used for a longer space of time, there is obscurity. This article shall give orientation.

A Medicament influences the complex feedback controlled systems of the organism analogous to a vehicle driver driving a car. Thus corresponding to the given situation it is sensible to increase medication during a crisis and reduce again at change for the better. Complications occur with this, if "the throttle is opened" or "the brake is blocked" in case of medication.

1 The Law by Weber and Fechner

1.1 In Words

The elaborations of Weber (1804–1891) and Fechner (founder of psychophysics or neurophysiology, respectively; 1801–1887) yield a logarithmic scale of nervous signals. Thus also for the sensation of brightness or loudness of a human being the subjective impression results, that a double signal followed by a quadruple signal would be equivalent amplifications.

1.2 Consequence for the Measuring of Loudness

As a consequence sound intensity is measured technically in *Phon*, a logarithmic unit, which according to DIN 45 630¹ gives a mean weight to the impressions of testing persons. The

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¹see [MS1998], experiment 9, page 9/5

technical measurement of *decibel* [dB] is done the following²:

$$z \text{ dB} = 10 \log \left(\frac{I_2}{I_1} \right) = 10 \log \left(\frac{p_2^2}{p_1^2} \right) = 20 \log \left(\frac{p_2}{p_1} \right) \quad (1)$$

with:

z : measuring number of decibel unit,

I_1 : reference intensity; when dealing with the unit [dB(A)] according to DIN 45 633³ via a normalized relating curvation as a sensible compromise of the instrumental signal,

I_2 : intensity of unknown measuring number,

p_1 : sound pressure corresponding to I_1 ,

p_2 : sound pressure corresponding to I_2 .

1.3 Consequence for the Measuring of Brightness

In physics brightness is not yet measured due to the law by Weber and Fechner, but as a multiple of the brightness of a normalized light source. Thus each doubling of the number of normalized light sources seems to the human eye always to be equivalent increase.

1.4 Determination of the Pitch in Music

1.4.1 Acoustical Impression

Also in music each doubling of frequency also always gives the impression of the same interval. The doubling of frequency (relation of frequency of oscillations 2 : 1) in European music tradition is called *octave*.

1.4.2 Rate of Equalized Intervals within the Octave

An equalized subdivision of the octave because of the law by Weber and Fechner must be oriented to equal *frequency relations* and not to equal *frequency differences*.

1.4.3 European Music Tradition

The mathematical formulation is given by the logarithm of the corresponding frequency relation, where in European music tradition meanwhile the unit *Cent* has been introduced to determine interval distances objectively:

$$1 \text{ Cent} := 2^{\frac{1}{1200}} \quad (2)$$

100 Cent are calculated the following:

$$100 \text{ Cent} = (1 \text{ Cent})^{100} = 2^{\frac{1}{12}} \quad (3)$$

²see [MS1998], experiment 9, page 9/3

³see [MS1998], experiment 9, page 9/5

Exactly this is an European *half-tone* in equally beating tune. Thus an octave has got 1200 Cent. A given frequency relation $\frac{f_2}{f_1}$ is calculated into Cent the following:

$$1200 \frac{\log\left(\frac{f_2}{f_1}\right)}{\log(2)} \text{ Cent.} \quad (4)$$

Since all logarithms are proportional to each other⁴, the calculation rule (4) depends on the fact only, that *each* logarithm of numerator and denominator is *the same*⁵.

1.4.4 Argumentative Avoiding of Logarithm

Since musicians not always dominate with logarithmic calculation, the following calculation way has become naturalized to avoid the word "logarithm":

Musical intervals are *added* by *multiplication* of their frequencies of oscillation⁶.

1.5 General Calculation Scheme for the Law by Weber and Fechner

What is possible in music is eventually always possible when discussing nervous impressions, namely the avoiding of the word "logarithm".

The following rate results by mathematical description of the law by Weber and Fechner:

$$\text{Rate} = 2^{\text{IntervalFactor}} \frac{\log\left(\frac{I_{\text{Actual}}}{I_{\text{Threshold}}}\right)}{\log(2)} = \left(\frac{I_{\text{Actual}}}{I_{\text{Threshold}}}\right)^{\text{IntervalFactor}} \quad (5)$$

Here $I_{\text{Threshold}}$ is the lowest perceptible intensity, the so-called *threshold dose*⁷. If I_{Actual} is less than $I_{\text{Threshold}}$, the nerves will detect no signal. The Rate in equation (5) therefore is useful for $I_{\text{Actual}} \geq I_{\text{Threshold}}$.

About the IntervalFactor must be set, what measuring number shall be supposed to be a doubling of intensity. In music (4) 1200 Cent correspond a doubling of frequency.

In pharmacology the intensities I_{Actual} and $I_{\text{Threshold}}$ are given by the amount (in mg) of active substance per time unit (e.g. per day). The threshold dose $I_{\text{Threshold}}$ has got an individual value for each patient.

2 Dragging Down of a Medicament

2.1 Principle

To reduce the dose of medicament continually by a uniform decrease of effect the necessity results that the dose (in mg/day) must be reduced *in each case* by the same factor.

Therefore as a primary calculation example the reduce in each case to the half is discussed.

⁴see [Rot1954], § 4.4, pages 25–26

⁵i.e. each logarithm having the same base

⁶i.e. their frequency relations

⁷Be careful: in [Bäu1994], section 3.3.3., page 64, this notion is used differently!

The following doses result as multiples of the original dose I_0 :

step	0	1	2	3	4	5	6	7	8	9	10
dose	I_0	$\frac{I_0}{2}$	$\frac{I_0}{4}$	$\frac{I_0}{8}$	$\frac{I_0}{16}$	$\frac{I_0}{32}$	$\frac{I_0}{64}$	$\frac{I_0}{128}$	$\frac{I_0}{256}$	$\frac{I_0}{512}$	$\frac{I_0}{1024}$

The presented table is dealed with by the entry *drag down*⁸ in the textbooks of medicine.

2.2 Misunderstandings

A reduction to $\frac{I_0}{2}$ and a further reduction to zero in opposite lead to a severe disturbance of health, because the reduction to zero can be done without risk only if the threshold dose $I_{\text{Threshold}}$ is already underbid. If the actual dose is yet above the threshold dose, a reduction can take place by a factor that turned out to be tolerable.

By discontinuation⁹ of medication always can be checked whether the threshold dose has yet been underbid:

- In case of a *relapse* the same must be catched by a short-term increase of dose, where the dragging down can start again at the change for the better.
- If there is no relapse, the threshold dose has been determined experimentally, whereby in the long run the tables of the pharmacologists can be corrected.

Since experiments on human beings are not allowed in Europe, pharmacological tables can also be incorrect or incomplete. Each patient of course has got the possibility to check by a well-considered self-experiment whether the information about the threshold dose for his medicament is correct.

A medicament should on principle be discontinued¹⁰ not before the impression has taken place that the taken dose is without effect.

2.3 Recommendation

The following reduction is recommended which represents about an exponential dragging down, whereby the start here is with 1000 mg/day:

step	0	1	2	3	4	5	6	7	8	9	10
dose [mg] day	1000	750	500	375	250	200	150	100	75	50	37.5
step	11	12	13	14	15	16	17	18	19	20	21
dose [mg] day	25	20	15	10	7.5	5.0	3.75	2.5	2.0	1.5	1.0

The given series can be realized by a *pill separator*¹¹ and by some pills of several dose.

The given recommendation can and should be varied individually, especially answering the question after what time a further reduction step will takes place.

⁸meaning an exponential reduction by a *constant* half-life

⁹i.e. abrupt omission

¹⁰i.e. abruptly being omitted

¹¹available in each chemist's shop

3 Control to Feedback Controlled Systems

3.1 Engineering Skill

Engineering skill knows about control to feedback controlled systems very well. Also control programs can be written, e.g. to start and turn off again a pump in dependence to the charge.

The property of each control to feedback controlled systems is the attention to the so-called *dead time*, namely the reaction time of the instrument to control actions. The space of time wherein control signals are sent thereby must not be shorter than the very dead time.

If the control takes place faster than allowed by the dead time, the result is system oscillation, i.e. an overmodulation of the system, which never can run optimized.

3.2 Control to a Car

When driving a car the reaction time of the driver usually is greater than the dead time of the vehicle. Thus the driver can and should always react as fast as possible.

The hand-brake or locking brake on principle is to be loosed completely during the drive, the accelerator pedal must not be pressed totally with the neutral gear. In driving school a driving style being adapted to the traffic situation is trained.

3.3 Medication of a Human Being

Even more complicated than driving a vehicle through the traffic is medication of a human being. The dead time of the organism thereby thoroughly can be much greater than the incorporation time of a medicament. Digestion can take place for some hours. Especially retard¹² pills own a large dead time.

Thus dealing with the correct reduction of medince results:

- More than one singular change of medication per day is too fast¹³.
- Medication for controlling versus an acute case is sensible and helpfull, but for permanence as senseless as a locked hand-brake during a drive.
- If after medication the situation turns for the better, the aimed dragging down can start.
- Also for some medical doctors mathematics of dragging down is too high, thus less useful methods of reduction or even discontinuation are used.

4 Final Remark

This essay is the trial to meet in a constructive way with the large ignorance of the topic dragging down of a medicament.

The author is grateful to tasks and improving suggestions.

¹²By construction of the pill the active substance is delivered continually for a longer time.

¹³An exception may be the direct injections into the vein.

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